SOLUTION OF A NONLINEAR HEAT CONDUCTION PROBLEM WITH BOUNDARY CONDITIONS OF THE FOURTH KIND

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UDC 681.142.334

We present a method for electrically modelling nonlinear contact heat-transfer problems both with and without taking into account the thermal conductivity of the contact layer.

To determine the temperature fields in bodies of complex configurations, as well as in the solution of other problems of field theory with complicated boundary conditions, wide use is being made of the method of electrical analogy to solve successfully both linear and nonlinear problems. In particular, analog methods are available for solving nonlinear heat conduction with boundary conditions of the types I-IV on ohmic resistance networks (R-networks) [1], based on Liebmann's method, the solution being obtained iteratively. References [2, 3] are directly concerned with modelling nonlinear problems of contact heat transfer.

The method of successive approximations is applied in [2] also, however, in contrast to the procedure in [1], only the boundary resistances are changed after each approximation, and not all the resistance of the R-network as was required in [1].

The solution given in [3] involved a linearization of the boundary conditions, followed by an introduction of new functions and a reassignment of the remaining boundary conditions. This method requires matching of the separate resistances of the conducting media employed for modelling the bodies in contact (or a corresponding selection of the parameters of the R-networks used in modelling with networks of ohmic resistances). The solution of nonlinear problems of field theory by RC-networks was, until recently, considered to be unrealizable in general, being possible only as a result of applying special transformations and using special devices to model nonlinear boundary conditions [4]. Unfortunately, the problem with boundary conditions of the fourth kind remained unsolved in view of its complexity and the need for a special approach.

In this paper we present a method for modelling a nonlinear contact heat-transfer problem, based on a combined use of passive models and apparata constructed on the principle of electronic modelling.

Since the apparata for modelling contact heat transfer, which is our basic concern here, are universal, i.e., they are equally available for solving both stationary and nonstationary problems, for simplicity we can, with no loss in generality, consider the stationary problem.

Assume that the thermal conductivity coefficients of two bodies in contact are functions of the temperature: $\lambda_1(t)$ and $\lambda_2(t)$.

Then the stationary heat conduction equations for these bodies may be written as follows:

$$\begin{gathered}\frac{\partial}{\partial x}\left[\lambda_1(t)\frac{\partial t}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_1(t)\frac{\partial t}{\partial y}\right]+\frac{\partial}{\partial z}\left[\lambda_1(t)\frac{\partial t}{\partial z}\right]=0,\\ \frac{\partial}{\partial x}\left[\lambda_2(t)\frac{\partial t}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_2(t)\frac{\partial t}{\partial y}\right]+\frac{\partial}{\partial z}\left[\lambda_2(t)\frac{\partial t}{\partial z}\right]=0.\end{gathered}\tag{1}$$

V. I. Lenin Polytechnical Institute, Khar'kov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 2, pp. 328-332, February, 1970. Original article submitted March 17, 1969.

Boundary conditions of the fourth kind, which equate temperatures and heat fluxes on the boundary, are usually written in the form:

$$\lambda_1(t)\left.\frac{\partial t}{\partial n}\right|_1=\lambda_2(t)\left.\frac{\partial t}{\partial n}\right|_2, \tag{2}$$

$$t_1=t_2. \tag{3}$$

However, in recent years the concept of contact heat transfer has been somewhat broadened (see, for example, [3]) to take into account the thermal conductivity of the contact layer formed of protrusions of roughness. In this case, the boundary condition (2) stays the same and the condition (3) is replaced by the following:

$$k(t_1-t_2)=-\lambda_1(t)\left.\frac{\partial t}{\partial n}\right|_1. \tag{4}$$

We apply to Eqs. (1)-(4) the integral transformations

$$\Phi=\int_0^t\lambda_1(t)\,dt;\qquad F=\int_0^t\lambda_2(t)\,dt. \tag{5}$$

The Eqs. (1) are thereby converted into Laplace equations, which may be simulated by passive models (R-networks or electrically conducting paper).

The boundary conditions (2)-(4) assume the form:

$$\left.\frac{\partial\Phi}{\partial n}\right|_1=\left.\frac{\partial F}{\partial n}\right|_2, \tag{6}$$

$$t_1(\Phi)=t_2(F), \tag{7}$$

$$k\left[t_1(\Phi)-t_2(F)\right]=-\left.\frac{\partial\Phi}{\partial n}\right|_1. \tag{8}$$

The electrical arrangement for the case of the boundary conditions (6)-(7) is shown in Fig. 1.

Between the boundary points of the two passive models 1 and 2 there is included a rheostat 3, which, along with the servomotor 4, the differential amplifier 5, and the two functional transformations 6 and 7, defines the following system. Control or variation of the resistance 3 proceeds so long as the equality (7) is not achieved, i.e., as long as the error signal put out by the differential amplifier is not equal to zero.

Since the functions $\lambda_1(t)$ and $\lambda_2(t)$ are, in general, distinct, the functions Φ and F may then so differ from one another that an alternate situation arises, wherein the current must be a flow from a point with a smaller potential to a point with a higher potential. This situation will prevail, for example, for a general direction of the current from model 1 to model 2, the function F on the boundary being larger than the

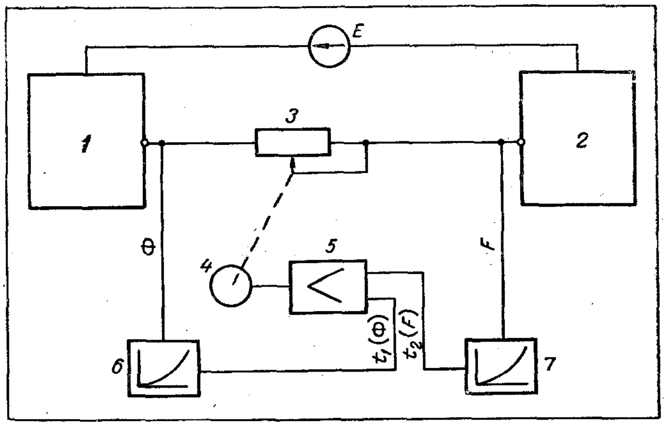

Fig. 1. Arrangement for achieving boundary conditions of the fourth kind.

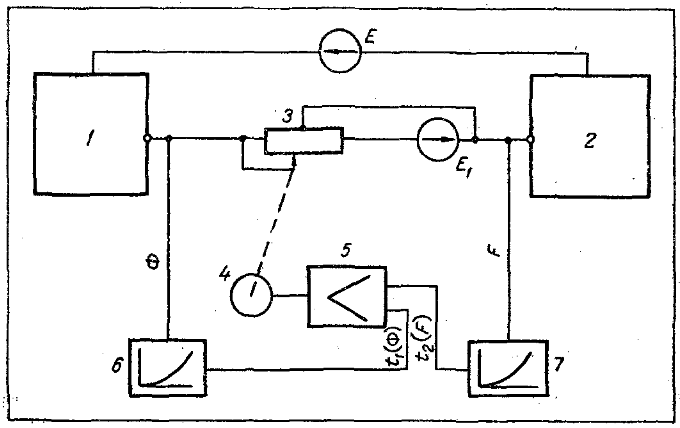

Fig. 2. Preliminary arrangement of an auxiliary source with controllable electromotive force.

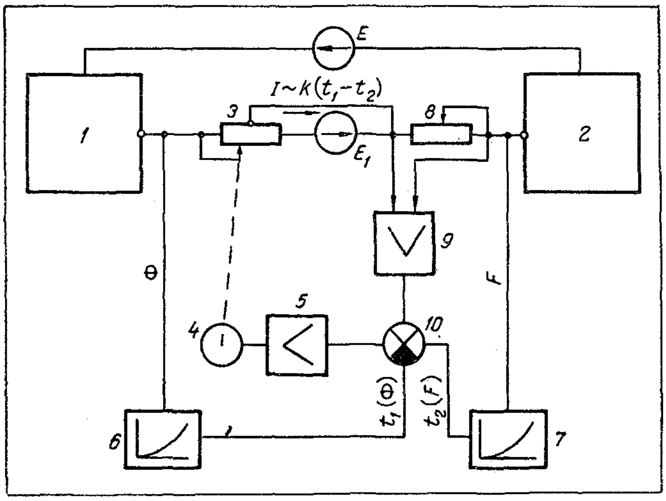

Fig. 3. Arrangement for taking into account the thermal conductivity of the contact layer.

boundary value of the function Φ. In this case the electrical arrangement becomes somewhat more complicated (Fig. 2) since an additional power supply E_1 is introduced with an electromotive force directed opposite to the electromotive force of the basic source E. Moreover the mechanism for handling the error signal and the control remains the same.

Up to a definite instant, namely up to the passage of the potentiometer indicator to its mean position the arrangements in Figs. 1 and 2 are equivalent. After the mean position is passed, the auxiliary source comes into play, where the electromotive force entering the main circuit depends on the position of the indicator: the further it passes the mean position, the larger the electromotive force.

In the case of the boundary conditions (6) and (8), i.e., when the thermal conductivity of the contact layer is taken into account, the circuit becomes more involved (Fig. 3) since the error signal is produced by comparing the current passing from the one model to the other with a quantity proportional to the left side of Eq. (8). This is achieved, in turn, by introducing with the resistance 3 a measuring resistance 8, whose voltage is fed to the input of the amplifier 9. The output of the amplifier 9 is connected with the adder–subtractor 10, into which are also fed the signals from the function converters 6 and 7, which convert the potentials of the boundary points to conform to the transformations (5).

Control of the resistance 3 is effected by the servomotor 4, whose amplifier is connected to the output of the adder–subtractor 10.

In order to provide for the case where the current must flow from a point of lower potential to a point of higher potential, both in the arrangement shown in Fig. 3 as well as in the previous arrangement, an auxiliary source E_1 is provided, which, from some instant on, also begins to partake in the control.

Thus, independent of the statement of the problem (with or without taking into account the thermal conductivity of the contact layer), it can be solved through analog devices using one scheme or another for handling the boundary conditions.

The solution of a nonlinear contact heat-transfer problem through the use of the arrangements proposed above differs advantageously, in our view, from, on the one hand, methods based on Liebmann's method, since it does not require recomputation and reassignment of all the resistances of the R-network after each approximation, and, on the other hand, the methods of [2, 3], since linearization of the boundary conditions is avoided and the results are obtained in one step without involving the method of successive approximations.

NOTATION

t	is the temperature;
λ	is the thermal conductivity;
α	is the heat-transfer coefficients;
x, y, z	are the Cartesian coordinates;
n	is the direction of outer normal to body surface.

Subscripts

1, 2 denote the first and second contacting bodies, respectively.

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